

Final Exam

All of your answers/solutions/computations must be written on the provided Answer Sheet.

Matching

Match the following vocabulary terms with the best description.

1. **A.** a cipher system that replaces the letters in a message with other characters
2. **B.** a prime number that can be expressed as one less than a power of two
3. **C.** a graph where every vertex is adjacent to every other vertex
4. **D.** an edge that begins and ends at the same vertex
5. **E.** a cipher system that scrambles the ordering of the letters in a message
6. **F.** the number of edges that start or end at a vertex
7. **G.** a pair of positive integers whose greatest common factor is 1
8. **H.** an edge that when removed will cause the graph to become disconnected
9. **J.** a graph where a path exists from every vertex to every other vertex
10. **K.** a positive integer that is only divisible by itself and the number 1
11. **M.** a pair of prime numbers whose difference is exactly 2

Multiple Choice

Read each question and then select the most correct response or responses.

7. In the graph shown at right, which of the following vertices is a cut vertex?
A. **B.** **C.**
8. The graph shown at right is planar.
A. True **B.** False **C.** depends
9. The graph shown at right is not complete. How many more edges need to be *drawn* to make the graph complete?
A. **B.** **C.**
10. The graph shown at right does not contain an Euler path. Which of the following edges, when added to the graph, will allow an Euler path?
A. **B.** **C.** **D.**
11. The number 123 is congruent to the number 62 in a modular 9 arithmetic system.
A. True **B.** False **C.** Depends **D.** Impossible to tell
12. Which of the following is not a triangular number?
A. **B.** **C.** **D.** **E.**
13. _____ proved that _____ .
A. **B.** **C.** **D.** **E.**
14. The number 2 is the smallest prime number.
A. True **B.** False **C.** Depends
15. In the context of Number Theory, $\pi(54) =$ _____.
A. **B.** **C.** **D.** **E.**

(continued on reverse side)

Free Response – Procedural/Computational

Answer the following being sure to show the work you did to arrive at your answer whenever possible.

- 16. Create a subgraph of the graph shown at right by applying Dijkstra’s algorithm starting from vertex P
- 17. Using only addition & multiplication, solve $42x + 16 \equiv 30 \pmod{11}$
- 18. The Highwired Cipher System is an affine cipher in mod 26 that is combined with backwards writing. Encode the word HELP using additive key 8 and multiplicative key 3. Be sure to clearly show each of the steps you take in the enciphering process.
- 19. George has constructed a graph with 7 vertices that have the following degrees: $___, ___, ___, ___, ___, ___, ____$
Beth says that such a graph is impossible. Who do you agree with and why?

20. Kevin decided to simplify his fashion life by devising a systematic way for choosing his daily attire. Starting January 1st of each year, he will wear the items listed first in the chart at right. On each subsequent day, he will wear the next item at the list. When he gets to the end of a particular list, he will move back to the top of that list.

Use modular arithmetic to predict Kevin’s attire on his birthday which is on June 3rd

Jan	Feb	Mar	Apr	May	Jun
31	28	31	30	31	30

Pants/Shorts	Shirt	Shoes
Cargo Pants	Red Polo Shirt	Doc Martins
Acid-Washed Jeans	Brown T-Shirt	Sandals
Khaki Pants	Green Dress Shirt	Skechers
	Blue Thermal Shirt	
	Gray T-Shirt	